

The Strengths and Weaknesses of the Subjective Interpretation of Probability

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A number of interpretations arose from the cyclical pattern of developing axioms to formalize a pre-formal understanding of probability. Philosophers of probability raise the question of what are probabilities and how can they be found; depending on the interpretation used, different answers emerge. A simple interpretation is the starting point for finding an answer to these questions. The interpretation should be precise, unambiguous, non circular, and well understood. For probability, additional criteria include admissibility, ascertainability and applicability (AAA) which will be defined later on. This paper will briefly mention the classical, logical, and frequency interpretations, then it will dive into the subjective interpretation of probability, and finally, it will argue the subjective interpretation's strengths and defend its weaknesses with respect to the criteria (AAA) outlined above in order to claim why it is the best interpretation.

First, in the classical interpretation, a probability is “the ratio of favorable cases to the number of equally possible cases.” [1] The probability of an event occurring is simply a fraction of the total number of possibilities. Laplace formulates the classical interpretation of probability as the ratio of events *of the same kind* to all possible cases. This interpretation is restricted to finite spaces and leads to various issues including paradoxes when measuring a probability in infinite spaces.

The logical interpretation attempts to “formalize the degree of support” [2] that we can derive from evidence to confirm a hypothesis. Similar to the classical interpretation, the logical interpretation's idea is that “probabilities can be determined a priori by an examination of the space of possibilities.” [2] This interpretation differs from classical because the probabilities of events do not have to be equally likely. Logical interpretations claim that probability implies that evidence E strongly supports hypothesis H but does not logically entail it. The issue of inductive reasoning ties closely with this interpretation but will not be discussed in this paper.

The frequency interpretation is split into finite relative frequency and hypothetical relative frequency. Finite relative frequency attaches probabilities to finite classes — i.e. given

a dataset of people who wear glasses or not, the probability of someone wearing glasses is simply the finite relative frequency (count of people who wear glasses in the data set over the total number of items in the data set). Hypothetical relative frequency allows for imagining an infinite set. For example, the probability of getting heads when flipping a coin is the value the limiting relative frequency of heads within all coin flips would be if we could flip the coin forever.

Finally, in the subjective interpretation the probabilities represent degrees of belief of rational agents. To be a valid belief “a rational agent is required to be logically consistent.” [2] (the axioms of probability must be followed). Each set of statements claimed by the agent is “logically consistent if they can all be true at the same time” [3] which essentially means that the set of beliefs can not contradict one another. Subjective Bayesians stop at this constraint while Objective Bayesians tack on that “rational probabilities [should be] constrained to the point of uniqueness by one’s evidence” [2] (Uniqueness Thesis). The Uniqueness Thesis holds that there is a unique set of probabilities assigned to a particular set of evidence.¹ The agent must also be “ethically neutral” meaning that the agent is indifferent to the outcome of a probability and whether it is true or false. The subjective interpretation of probability is limited primarily by how it constrains belief and credence.

By constraining beliefs by the probability axioms, the probabilist avoids falling for the Dutch Book. The Dutch Book Argument states that if the probability axioms are not satisfied then “there is a set of bets... that guarantee a net loss.” [4] In other words, if an inaccurate assumption is made about the probability of an event then the intermediary profits. Given the restriction of the axioms, we can make a few claims about the validity of subjective interpretation.

The subjective interpretation of probability satisfies the admissibility, ascertainability and applicability criteria. First, a probability is admissible “if the meanings assigned to the primitive terms in the interpretation transform the formal axioms, and consequently all the

¹For the remainder on the paper, we will assume a position half way between the two extremes.

theorems, into true statements”. [2] The argument that allows the subjective interpretation to satisfy admissibility is given by the self applied constraint of the probability axioms to avoid being Dutch Booked.

Ascertainability simply states that we must be able to find the probabilities. This requirement for interpretability essentially means that the concept of probability provides no value or new information if it is impossible to discover the probabilities of an event. This condition is met if we assume that we have met the prior criteria that our agent is rational and that the full extent of their beliefs and corresponding probabilities have been taken into account. We obtain these prior probabilities through prior knowledge derived from an observed frequency or the knowledge of subject matter experts (i.e. if the agent has no knowledge on a subject, it should adopt the expert probability assignment as its probability).

The third criteria of a valid interpretation is applicability. This is broad but can be narrowed down to focus on the applicability to frequencies (probability should be related to limiting frequency), rational beliefs (the role probabilities play on constraining beliefs), rational decisions, and science. [2] We note that the subjective interpretation satisfies the applicability to frequencies as it takes into account frequencies but *also* includes beliefs. It satisfies rational beliefs and rational decisions because we constrain the beliefs by assuming a fully rational agent that does not care whether a hypothesis will be true or false (thus unbiased) and by allowing the probability axioms to be true. Finally, it is applicable to science because it allows us to include rational beliefs in science through probability.

A Bayesian skeptic may question whether subjectivity should be included in science. Lindley argues that, in fact, the scientific method is subjective. He makes the claim that “the scientific method consists in expressing your view of your uncertain world (hypothesis) in terms of probability, performing experiments to obtain data, and using that data to update your probability and hence your view of the world.” [5] With frequentist methodology, you can not *technically* make any claim of significance (in the colloquial use of the word) about your results. Thus, the ability to incorporate beliefs into probability, allows us to properly

utilize the scientific method to make inferences about our uncertain world.

A firm believer of the superiority of an alternative interpretation may object to the arguments above. For one, the issue of whether or not the Dutch Book Argument can be undone while it is in the process may be raised — i.e. can an agent be induced into continuing to take all of the bets in the book or will the agent stop when they realized they are being Dutch Booked. [4]

To this, the subjective probabilist may respond that there exists a converse to the Dutch Book called the Czech Book. [6] This book takes the play book of the Dutch Book and replaces the word ‘loss’ with the word ‘gain’. The Dutch Book states: “If you violate probability theory, there exists a set of bets, each of which you consider fair-*or-favourable*, which collectively guarantee your loss” [6] and the Czech Book states: “If you violate probability theory, there exists a set of bets, each of which you consider fair-*or-favourable*, which collectively guarantee your gain” [6] which imply that “incoherence cannot be condemned as irrational simply by citing the possibility of sure losses”. [4] The key is that an individual will not make the bet that is fair but rather will take bets that are likely in their favor. Thus a coherent agent that discovers that they are being Dutch Booked *can* stop.

Another issue with subjective probability that may be observed skeptically is that the interpretation assumes that a large proportion of the uncertainty is epistemic. Because this interpretation incorporates the beliefs of the agent, it isn’t clear whether the errors arise from lack of information or from inherent uncertainty.

To this point the subjective probabilist may respond with a counterexample against other probability interpretations. For example, the claim “there is a 0.01 probability that my friend Bob will run for president” is reasonable under the subjective interpretation. In this example, it is unclear how relative frequency would apply. If Bob has never run for president in the past, then the relative frequency would be undefined rendering the frequency interpretation of probability invalid. A similar argument could be made for the classical interpretation. How can we assign a probability to Bob running for president? Would the probability of

Bob becoming president be the same as, for example, my friend Sara running for president? As for propensity, does Bob have some sort of intrinsic characteristic about him that would make the probability that he runs for president a certain value that differs from another individual? [7] While it is unclear what the specific cause of the uncertainty is in some parts of the subjective interpretation, it is still able to assign probabilities given additional prior information from a rational agents knowledge and beliefs (which, as mentioned above, could potentially be a frequency - note: these are not epistemic).

The final argument against subjective probability in this paper is about whether or not a belief can actually be assigned a number. To this argument a subjective probabilist can respond by stating that the degrees of belief can themselves be interpreted in different ways, as long as they are consistent with the axioms (and theorems) of probability. Individuals may act as if they have degrees of belief that conform to probability calculus and “act to maximize utility” [8] or perhaps *should* act as if they have degrees of belief that do. Additionally, degrees of belief are said to be graded attitudes of human agents [9]. If a probability is a value between 0 and 1 and there can theoretically be infinite probabilities for a certain event, it should be possible to map another infinite set, i.e. degrees of belief, 1 to 1 onto it. Admittedly, the methodology for finding precisely the correct mapping isn’t clearly defined, however, theoretically there appears to be no reason that a belief can not be mapped to a probability.

Each interpretation is closely tied to philosophy. While each one raises different philosophical issues, the subjective interpretation of probability attempts to answer questions about existence by assigning degrees of belief to physical events. The subjective interpretation therefore closely ties to the philosophical interpretation of belief. In this interpretation, a probability is not a property of the world but rather a property of an individual’s information of the world. While quantifying belief can be a challenging task and subjective probabilists turn to approximation when doing so, it is valuable to assign beliefs to probabilities because it allows the incorporation of prior information. The long running debate about

the interpretations are not about finding the interpretation of probability but rather about finding one that is best for applications of probability. [10] Because the subjective interpretation allows for the incorporation of prior knowledge, it is wise to use this interpretation for external applications.

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